國立臺北大學 112 學年度日間學士班轉學生招生考試試題

學制系級：統計學系日間學士班 3 年級

科　　目：機率概論

☐可　☑不可 使用計算機

1. (25%) The random variable X has the moment generating function

   $$M(t) = \sum_{x=1}^{5} \frac{x}{c} e^{tx}$$

   (a) Find the unknown constant c.

   (b) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

   (c) Sketch the C.D.F. of X, $F_X(x)$.

   (d) Let $U = F_X(X)$. Find the C.D.F. of U.

2. (25 %) An urn contents 10 balls, numbered from 1 to 10. For each game, the player will select 2 balls and will win the game, if the sum of the numbers on the balls is 10. Let X be the number of games that the player needs to play for 1st win.

   (a) Find the p.m.f. of X and $\mathrm{E}(X)$ if the 2 balls are selected with replacement.

   (b) Find the p.m.f. of X and $\mathrm{E}(X)$ if the 2 balls are selected without replacement.

3. (30 %) Let random variables X and Y have the joint p.d.f.

   $$f(x,y) = cxy^3, \quad 0 < y < x < 1$$

   (a) Find the unknown constant c.

   (b) Find the marginal p.d.f.s of X and Y.

   (c) Calculate $\mathrm{E}(X)$, $\mathrm{E}(Y)$, $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$ and $\mathrm{Cov}(X)$.

4. (20 %) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with p.d.f.

   $$f(x) = cx(1-x), \quad 0 < x < 1$$

   (a) Find the unknown constant c.

   (b) Find the moment generating function of $X_i, i = 1,2, \ldots, n$.

   (c) Let $\bar{X}_n = (X_1 + X_2 + \cdots + X_n)/n$. Find the moment generating function of $\bar{X}_n$.

試題隨卷繳交