

國立臺北大學 113 學年度日間學士班轉學生招生考試試題

學制系級：金融與合作經營學系日間學士班 2 年級

科 目：統計學

第1頁 共1頁

可 不可 使用計算機

- 1、(20%) A vendor of milk products produces and sells low-fat dry milk to a company that uses it to produce baby formula. In order to determine the fat content of the milk, both the company and the vendor take a sample from each lot and test it for fat content in percent. Ten sets of paired test results

Lot Number	1	2	3	4	5	6	7	8	9	10
Company Test Results (X)	0.50	0.58	0.90	1.17	1.14	1.25	0.75	1.22	0.74	0.80
Vendor Test Results (Y)	0.79	0.71	0.82	0.82	0.73	0.77	0.72	0.79	0.72	0.91

Let $\mu_D = \mu_X - \mu_Y$ denote the mean of the differences between X and Y. Test $H_0: \mu_D = \mu_X - \mu_Y = 0$ against $H_1: \mu_D = \mu_X - \mu_Y > 0$ using t-test for the differences. Let $\alpha = 0.05$. (All values are rounded to the third decimal place.) (Hints: $t_{0.95}(9) = 1.833$; $t_{0.975}(9) = 2.262$; $t_{0.95}(10) = 1.812$; $t_{0.975}(10) = 2.228$, where $P(t < t_{1-\alpha}(r)) = 1 - \alpha$ for a t random variable with a freedom r)

- 2、(20%) Each of four persons fires one shot at a target. Let C_k denote the event that the target is hit by person k , $k = 1, 2, 3, 4$. If C_1, C_2, C_3, C_4 are independent and if $P(C_1) = P(C_2) = 0.7$, $P(C_3) = 0.9$, and $P(C_4) = 0.4$, compute the probability that (a) all of them hit the target; (b) exactly one hits the target; (c) no one hits the target; (d) at least one hits the target.

- 3、(20%) A random sample X_1, X_2, \dots, X_n of size n is taken from a Poisson distribution with a mean of λ , $0 < \lambda < \infty$.
- (a) (15%) Find the maximum likelihood estimator for λ , and prove it.
- (b) (5%) Let X equal the number of flaws per 100 feet of a used computer tape. Assume that X has a Poisson distribution with a mean of λ . If 40 observations of X yielded 5 zeros, 7 ones, 12 twos, 9 threes, 5 fours, 1 five and 1 six, find the maximum likelihood estimate of λ .

- 4、(20%) Some dentists were interested in studying the fusion of embryonic rat palates by using a standard transplantation technique. When no treatment (vitamin A) is used, the probability of fusion approximately equals 0.89. They would like precisely to estimate p , the probability of fusion, when vitamin A is lacking.
- (a) (10%) Let y denote the number of fusions in the sample. How large a sample n of rat embryos is needed for $y/n \pm 0.10$ to be a 95% confidence interval for p ?
- (b) (10%) If $y = 44$ out of $n = 60$ palates showed fusion, give a 95% confidence interval for p . (All values are rounded to the third decimal place.)

- 5、(20%) Let the random variables X and Y have the joint p.d.f. $f(x, y) = (2/\theta^2)\exp(-(x+y)/\theta)$, $0 < x < y < \infty$, zero elsewhere.
- (a) (10%) Show that the mean and variance of Y are, respectively, $3\theta/2$ and $5\theta^2/4$.
- (b) (10%) Show that $E(Y|x) = x + \theta$. In accordance with the theory, the expected value of $X + \theta$ is that of Y , namely, $3\theta/2$, and the variance of $X + \theta$ is less than that of Y . Show the variance of X is in fact $5\theta^2/4$.

(Hint: $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$, and $\Gamma(n) = (n-1)!$ for $n = 1, 2, \dots$, $\Gamma(1) = 0! = 1$)