

國立臺北大學 105 學年度日間學士班暨進修學士班轉學生招生考試試題

系 別：電機工程學系日間學士班 3 年級

考試時間：80 分鐘

科 目：工程數學

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可 不可 使用計算機

1. Solve the initial value problem for the following first-order ordinary differential equations.

(a) (5%) $\frac{dy}{dt} = -0.5y, y(0) = 3$

(b) (5%) $\frac{dy}{dt} = 50 - 0.01y, y(0)=100$

(c) (5%) $\frac{dy}{dx} = (x + y - 2)^2, y(0)=2$

2. Solve the initial value problem for the following second-order ordinary differential equations.

(a) (5%) $y'' + y = 0, y(0) = 3, y'(0) = -0.5$

(b) (5%) $y'' + y' + 0.25y = 0, y(0) = 3, y'(0) = -3.5$

(c) (5%) $x^2y'' + xy' + 9y = 0, y(1) = 0, y'(1) = 2.5$

3. (15%) Solve the initial value problem by using the Laplace transform.

$$y'' + 4y' + 3y = f(x), y(0) = y'(0) = 0,$$

where $f(x) = 5x$, if $0 < x < 2$ and $f(x) = 10$, if $2 < x$.

4. (10%) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at point $P: (2, 1, 3)$ in the direction of $\mathbf{a} = [1, 0, -2]$.

5. Consider a force field $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$, and answer the following questions:

(a) (5%) Show that the force is conservative.

(b) (5%) Find the potential function $\phi(x, y, z)$ for this force field.

(c) (5%) Calculate the work done by this field upon a particle moving from position $(1, -2, 1)$ to position $(3, 1, 4)$.

6. (15%) Find the Fourier series of the following periodic function $f(x)$ with period 2π .

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

7. (15%) Integrate counterclockwise

$$\oint_C (z + z^{-1})dz,$$

where C is the unit circle.