國立臺北大學 103 學年度學士班暨進修學士班轉學生招生考試試題

系　　別：電機工程學系、通訊工程學系學士班 3 年級　考試時間：80 分鐘

科　　目：工程數學　　第 1 頁 共 1 頁

☐可 ☑不可 使用計算機

1. (10%) Let $a_n = i/2^{3n}$ and $b_n = 1/2^{3n+1}$. Is the following series convergent or divergent?

   $$a_o + b_o + a_1 + b_1 + \cdots = i + \frac{1}{2} + \frac{i}{8} + \frac{1}{16} + \frac{i}{64} + \frac{1}{128} + \cdots$$

2. (10%)Find all Taylor and Laurent series with center 0 of

   $$f(z) = \frac{-2z+3}{z^2-3z+2}.$$

3. (10%)Integrate the following function counterclockwise around C. Show the details.

   $$\frac{15z+9}{z^3-9z}, C: |z| = 4$$

4. (10%)Use complex integration method to find

   $$\int_{-\infty}^{\infty} \frac{dx}{x^2 - 4ix}$$

5. (10%) Prove the following Fourier transform operation.

   $$v(t-\tau) \xrightarrow{F} e^{-j2\pi f\tau}\overline{V}(f)_{\mathrm{V}}$$

6. (8%) Solve

   $$[y^2 - e^x \sin(e^x + 1)]dx + 2xydy = 0.$$

7. (8%) Solve

   $$(x + 2y + 3)dx + (x - 2y + 4)dy = 0.$$

8. (8%) Find the orthogonal trajectories of $(x-c)^2 + y^2 = c^2$.

9. (8%) Solve

   $$\frac{d^2y}{dx^2} + y = x + \sec x.$$

10. (8%) Find the Fourier series of the period function $f(x) = x + \pi, |x| < \pi$.

11. (10%) Solve the following initial value problem by the Laplace transform.

    $$\begin{cases} y_1'(x) = y_1(x) - 2y_2(x) + 1, \\ y_2''(x) = -3y_1(x) - 2y_2(x), \\ y_1(0) = y_2(0) = y_2'(0) = 0. \end{cases}$$

試題隨卷繳交