

國立臺北大學 106 學年度學士班暨進修學士班轉學生招生考試試題

系別：資訊工程學系 3 年級(學士班)

考試時間：80 分鐘

科目：線性代數

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可 不可使用計算機

1. A matrix M is called a skew symmetric if $M^T = -M$. Prove that if A is an $n \times n$ skew symmetric matrix and n is odd, then A is singular.(12%)

2. Find the standard matrix representation for the following linear transformation L : L reflects each input vector x about the line $x_2=x_1$ and then projects it onto the x_1 axis.(12%)

3. Let M be a 2×2 matrix with $\text{tr}(M)=\sqrt{3}$ and $\det(M)=-2$. Let α_1 and α_2 be eigenvalues of M . What is $|\alpha_1-\alpha_2|$? (12%)

4. Find a matrix B such that $B^3 = A$ where $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & -1 \end{bmatrix}$ (14%)

5. Project the vector $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ onto the plane contains $a_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $a_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(1)Find the 3 by 3 projection matrix P onto the plane of a_1 and a_2 . (5%)

(2)Find the projection p of b onto the plane of a_1 and a_2 . (5%)

(3)Find the error vector $e = b - p$. (5%)

(4)Find a vector whose projection onto the plane of a_1 and a_2 is the zero vector. (5%)

6. Give the following matrix A .

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 2 & 1 & 7 \end{bmatrix}$$

(1)Find the column space of A . (5%)

(2)Find the complete solution of $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. (10%)

(3)Give a basis for the nullspace of A . (5%)

7. Invert the matrix $B = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ by the Gauss-Jordan method starting with $[B \ I]$. (10%)