

國立臺北大學 115 學年度日間學士班轉學生招生考試試題

學制系級：統計學系日間學士班 2、3 年級

科 目：微積分

第1頁 共2頁

可 不可 使用計算機

1. (10%) If the functions are given, answer the following questions:

$$f(x) = x^2 - x - \ln x$$

- (a) (2%) Find the critical numbers of the function.
- (b) (2%) Find the local maximum and minimum values of the function.
- (c) (2%) Find the intervals of concavity and the inflection points.
- (d) (4%) Sketch the curve.

2. (15%) Evaluate the definite integral.

(a) (5%) $\int_0^1 \cos(\pi x/2) dx$

(b) (5%) $\int_1^2 (\ln x)^2 dx$

(c) (5%) $\int_0^1 \frac{2}{2x^2+3x+1} dx$

3. (15%) Evaluate the indefinite integral.

(a) (5%) $\int e^x \sqrt{1+e^x} dx$

(b) (5%) $\int (x^2 + 2x) \cos x dx$

(c) (5%) $\int e^{2x} \sin(3x) dx$

4. (8%) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) (4%) $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$

(b) (4%) $\int_1^{\infty} \frac{x+1}{\sqrt{x^4-x}} dx$

5. (6%) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = \sqrt[3]{x}, \quad y = 0, \quad x = 1 \quad \text{about the } y\text{-axis}$$

試題隨卷繳交

接背面

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第2頁 共2頁

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6. (6%) Determine whether the sequence converges or diverges. If it converges, find the limit.

(a) (3%) $a_n = \frac{(-1)^n}{2\sqrt{n}}$

(b) (3%) $a_n = \frac{\cos^2 n}{2^n}$

7. (6%) Find the radius of convergence and interval of convergence of the series.

(a) (3%) $\sum_{n=0}^{\infty} \frac{(x)^n}{n!}$

(b) (3%) $\sum_{n=1}^{\infty} n! (2x - 1)^n$

8. (8%) Find the critical points, the local maximum or minimum values, and the saddle points of the function.

$$f(x, y) = x^3 - 12xy + 8y^3$$

9. (6%) Let the equation be $e^z = xyz$. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

10. (20%) Calculate the double integral.

(a) (5%) $\int_0^1 \int_0^{e^y} \sqrt{1 + e^y} dx dy$

(b) (5%) $\iint_D x^3 dA, D = \{(x, y) \mid 1 \leq x \leq e, 0 \leq y \leq \ln x\}$

(c) (5%) $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dy dx$

(d) (5%) $\iint_D x y^2 dA, D$ is bounded by $x = 0$ and $x = \sqrt{1 - y^2}$

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