

國立臺北大學 115 學年度日間學士班轉學生招生考試試題

學制系級：資訊工程學系日間學士班 3 年級

科 目：線性代數

第1頁 共1頁

可 不可使用計算機

1. (20%) Completing the following traffic flow by using matrix multiplication. Calculate the number of cars leaving the network

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \text{ given the number of cars entering the network } \vec{x} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix}.$$

2. (20%) Let A be an $n \times n$ matrix. Use elementary row operations to transform $[A \ I_n]$ to $[R \ B]$, where R is a matrix in reduced row echelon form. If $R = I_n$, in which case A is invertible, show that $B = A^{-1}$.

(Hint: Using the fact that there exists an invertible matrix P such that $PA=R$)

3. (40%) Prove that: An $n \times n$ matrix A is diagonalizable **if and only if** there is a basis for \mathbb{R}^n consisting of eigenvectors of A . Specifically, $A = PDP^{-1}$ **if and only if** the columns of P are the basis of \mathbb{R}^n and the diagonal entries of D are eigenvalues corresponding to the respective columns of P .

4. (20%) Let A be any $m \times n$ matrix. Prove that $A^T A$ and A have the same null space.

(Hint: Let \vec{v} be a vector in \mathbb{R}^n such that $A^T A \vec{v} = 0$. Observe that $A^T A \vec{v} \cdot \vec{v} = A \vec{v} \cdot A \vec{v} = 0$)

試題隨卷繳交