國立臺北大學 104 學年度學士班暨進修學士班轉學生招生考試試題

系　　別：電機工程學系、通訊工程學系學士班 3 年級　考試時間：80 分鐘

科　　目：工程數學　　第 1 頁 共 1 頁

☐可 ☑不可 使用計算機

1. (8%) Solve the following ODE

$$(x^2 + 3y^2)dx - 4xydy = 0$$

where y is a function of x.

2. (12%) Let $y = (A + Bx)e^{ax} + Ce^{2x}$ be the general solution to the following linear constant-coefficient ODE

$$y''' + py' + qy = 0.$$

Solve a, p and q.

3. (12%) (a) Show that $y = e^x$ is a solution to the following ODE

$$(x - 1)y'' - xy' + y = 0.$$

(b) Find a basis of solutions of the above ODE.

4. (8%) Let $f(x)$ be a periodic function with period 2π and defined by

$$f(x) = \begin{cases} x + \pi, & |x| \le \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} < |x| \le \pi. \end{cases}$$

for $-\pi \le x \le \pi$. Find the Fourier series of $f(x)$.

5. (10%) Solve the following initial value problem by using the Laplace transform.

$$y_1'(x) + y_2(x) = x, \quad y_1(x) + y_2'(x) = 0,$$
$$y_1(0) = 1, \quad y_2(0) = 0$$

6. (10%) Please answer the following questions in details.

(a) (5%) What are Cauchy-Riemann Equations?

(b) (5%) What is the purpose of Cauchy-Riemann Equations?

7. (10%) Is the following function analytic?

(a) (5%) $f(z) = e^{-x} \cos y - i\, e^{-x} \sin y$

(b) (5%) $f(z) = Arg\, z$

8. (10%) **Integrate counterclockwise. Show the details.**

$\oint_C \frac{z}{z^2+4z+3} dz$, C the circle with center -1 and radius 2.

9. (10%) **Integrate counterclockwise. Show the details.**

$\oint_C \frac{e^z}{ze^z - 2iz} dz$, C:$|z| = 0.6$

10. (10%) Find the principal value

$$\text{pr. v.} \int_{-\infty}^{\infty} \frac{dx}{(x^2 - 3x + 2)(x^2 + 1)}.$$

試題隨卷繳交