國立臺北大學 112 學年度日間學士班轉學生招生考試試題

學制系級：資訊工程學系日間學士班 3 年級

科　　目：線性代數

☐可　☑不可使用計算機

1. Give the following matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & -1 \\ -1 & 2 & 5 \end{bmatrix}.$$

(a) (12%) Compute the LU factorization of **A**.

(b) (12%) Find the bases for the four fundamental subspaces of **A**, namely, the nullspace $N(\mathbf{A})$, the nullspace $N(\mathbf{A}^T)$, the range $R(\mathbf{A})$, and the range $R(\mathbf{A}^T)$, respectively.

(c) (6%) Verify that **A** satisfies the fundamental subspaces theorem.

2. Let L: $\mathbf{R}^2 \rightarrow \mathbf{R}^3$ be $L(\mathbf{x}) = (x_2, x_1 - x_2, x_2 - x_1)^T$.

(a) (10%) Show that L is a linear transformation.

(b) (10%) Give the following two ordered bases: $E = \{\mathbf{u}_1, \mathbf{u}_2\}$ where $\mathbf{u}_1 = (2, 1)^T$, $\mathbf{u}_2 = (1, 3)^T$, and $F = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ where $\mathbf{v}_1=(1, 0, 0)^T$, $\mathbf{v}_2=(1, 1, 0)^T$, $\mathbf{v}_3=(1, 1, 1)^T$. Find the matrix representation of L with respect to the ordered bases E and F.

3. Let $\mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}$.

(a) (10%) Solve the least squares problem $\mathbf{Bx} = \mathbf{b}$ for $\mathbf{b} = (1, 3, 2, 4)^T$.

(b) (10%) Compute the Gram-Schmidt QR factorization of the matrix **B**.

4. (20%) Diagonalize the matrix $\mathbf{C} = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{bmatrix}$.

5. (10%) Let **D** be an $m \times n$ matrix of rank n. Is the matrix $\mathbf{D}^T\mathbf{D}$ symmetric positive definite? Explain your answer.

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