

國立臺北大學 107 學年度日間學士班暨進修學士班轉學生招生考試試題

系 別：資訊工程學系日間學士班 3 年級

科 目：線性代數

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可 不可使用計算機

1. (10%) Use the elementary row operations to find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$

2. (10%) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$ Find an upper triangular matrix U and a lower triangular matrix L such that $A = LU$.

3. (10%) The following matrix is the adjoint matrix of A : $\text{adj } A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & -2 & -1 & 2 \end{bmatrix}$. Please calculate $\det(A)$.

4. Let $Ax = b$ be a linear system whose augmented matrix $(A|b)$ has reduced row echelon form $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & -2 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$.

(a)(10%) Find all solutions to the system.

(b)(10%) If $a_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$ and $a_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix}$

determine b

5. Define a mapping from \mathbb{R}^3 to \mathbb{R}^2 : $L(x) = (2x_2+5x_3, 3x_1+x_2)^T$.

(a)(5%) Show that the mapping L is a linear transformation.

(b)(5%) Find a matrix A such that $L(x) = Ax$ for each $x \in \mathbb{R}^3$.

6. Give the following system $Bx = b$ where

$$B = \begin{bmatrix} 1 & 1 \\ 2 & -3 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

(a)(10%) Find the least squares solution \hat{x} for the system $Bx = b$.

(b)(5%) Determine the projection $p = B\hat{x}$.

(c)(5%) Calculate the residual $r(\hat{x})$.

7. Give the following matrix C.

$$C = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

(a)(10%) Find the eigenvalues and the corresponding eigenvectors for the matrix C.

(b)(5%) Factor the matrix C into a product $XD X^{-1}$, where D is diagonal.

(c)(5%) Use the $XD X^{-1}$ factorization in (b) to compute C^{100} .