

國立臺北大學 105 學年度日間學士班暨進修學士班轉學生招生考試試題

系 別：資訊工程學系日間學士班 3 年級

考試時間：80 分鐘

科 目：線性代數

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可 不可 使用計算機

1. (10%) Let $u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Let L be the linear transformation defined by $L(x_1, x_2, x_3) =$

(x_1+x_2, x_1-x_3) . Find the matrix A representing L with respect to the bases $\{u_1, u_2, u_3\}$ and $\{b_1, b_2\}$.

2. (12%) Find the least squares solution of the following system:

$$\begin{aligned}x_1 + x_2 + x_3 &= 4 \\-x_1 + x_2 + x_3 &= 0 \\-x_2 + x_3 &= 1 \\x_1 + x_3 &= 2\end{aligned}$$

3. (12%) Find the eigenvalues and the corresponding eigenspaces for the following matrix:

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

4. (16%) Factor the matrix A into a product $XD X^{-1}$, where D is diagonal and $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & -1 \end{bmatrix}$.

(4%) (a) X , (4%) (b) D , (4%) (c) X^{-1} . (4%) (d) Find a matrix B such that $B^3=A$

5. (18%) Compute L and U for the following matrix A :

$$A = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 5 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}.$$

6. (16%) Find the nullspace of the following matrix B :

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

7. (16%) Compute the determinant of the following matrix C :

$$C = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 14 & 0 & 0 \\ 0 & 16 & 1 & 0 \\ 0 & 18 & 0 & 1 \end{bmatrix}.$$