國立臺北大學108學年度日間學士班轉學生招生考試試題

學制系級：統計學系日間學士班2年級

科　　目：微積分

☐可 ☑不可使用計算機

1. Let $f(x)$ be a polynomial of degree 3.

   (a) (10%) If $f(1)=18$ is a relative maximum and $f(-3)=-14$ is a relative of minimum f, please find $f(x)$.

   (b) (5%) Find the value of k so that $(k, f(k))$ is a point of inflection of $f(x)$.

2. Let $f(x)=\begin{cases}\dfrac{\ln x}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$.

   (a) (7%) Show that f is continuous at 1.

   (b) (8%) Show that f is differentiable at 1.

3. (a) (6%) Let $\dfrac{d}{dx}f(x)=e^{-x^2}$. Find $\dfrac{d}{dx}f(\sqrt{x})$, where $x>0$.

   (b) (7%) Let $f(x)=x^3+x+1$. Find $(f^{-1})'(3)$.

   (c) (7%) Find $\dfrac{d}{dx}x^x$.

4. Let $f(x,y)=\ln\sqrt{x^2+y^2}$.

   (a) (5%) Find the level curve of $f(x,y)=0$.

   (b) (7%) Find $\nabla f(1,-2)$, the gradient of $f(x,y)$ at $(1,-2)$.

   (c) (8%) Show that $f_{xx}(x,y)+f_{yy}(x,y)=0$.

5. Evaluate the following integrals.

   (a) (8%) $\displaystyle\int\frac{x+1}{x^3+x}dx$

   (b) (7%) $\displaystyle\int_0^\infty x^2e^{-2x}dx$

   (c) (8%) $\displaystyle\iint_R xydA$, where R is the region bounded by $y=x^2$ and $y=\sqrt{x}$.

   (d) (7%) $\displaystyle\iint_R e^{x^2+y^2}dA$, where $R=\{(x,y)\in\mathbb{R}^2\mid x^2+y^2\le 1\}$.

試題隨卷繳交