

國立臺北大學 107 學年度日間學士班暨進修學士班轉學生招生考試試題

系 別：統計學系日間學士班 2、3 年級

科 目：微積分

第1頁 共1頁

可 不可使用計算機

1. Let  $f(x) = \frac{x^3}{x^2 - 1}$ ,  $x \neq \pm 1$ .

- (a) (7%) Find all critical values of  $f$ .  
 (b) (6%) Find  $f''(x)$ , the second derivative of  $f$ .  
 (c) (6%) Find all relative extrema of  $f$ .  
 (d) (6%) Find all points of inflection of  $f$ .

2. Apply that  $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$  to evaluate the following improper integrals.

- (a) (7%)  $\int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$   
 (b) (8%)  $\int_0^{\infty} x^{\frac{3}{2}} e^{-x} dx$

3. Let  $f(x) = \frac{1}{1+x^2}$ ,  $0 \leq x \leq 1$  and  $P = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$  be a partition of  $[0, 1]$ .

- (a) (7%) Calculate the upper sum  $U(f, p)$  and the lower sum  $L(f, p)$  of  $f$  over the partition  $P$ . Rounded the answers to the third decimal place.

- (b) (6%) Approximate  $\int_0^1 \frac{1}{1+x^2} dx$  by the Simpson's rule with  $n=4$  and round to the third decimal place.

- (c) (7%) Evaluate  $\lim_{n \rightarrow \infty} \left[ \sum_{i=1}^n \frac{n}{n^2 + i^2} \right]$ .

4. Let  $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

- (a) (7%) Find  $f_x(0, 0)$  and  $f_y(0, 0)$ .  
 (b) (8%) Explain  $f$  is differentiable at  $(0, 0)$  or not.

5. Evaluate the following integrals.

(a) (7%)  $\int_1^{\infty} \frac{\ln x}{x^2} dx$

(b) (8%)  $\iint_R \frac{y}{1+x^2} dA$ , where  $R$  is the region bounded by  $y=0$ ,  $y=\sqrt{x}$ ,  $x=4$ .

(c) (10%)  $\iint_R \frac{xy}{\sqrt{1+x^2y^2}} dA$ , where  $R$  is the region bounded by  $xy=1$ ,  $xy=3$ ,  $y=x$ ,  $y=2x$ , by the setting  $u=xy$ ,  
 $v = \frac{y}{x}$ .

試題隨卷繳交