

國立臺北大學 114 學年度碩士班一般入學考試試題

系(所)組別：都市計劃研究所

科目：微積分

第1頁 共1頁

☐可 ☒不可使用計算機

1. (10%) Suppose the function $f(x) = \frac{1}{3}x^3 - ax^2 + bx + c$ has extreme values at $x = 1, 2$ and $f(1) = 5$. Find a, b, c .

2. (10%) Find the local extrema of $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 - x^3 + 4x^2 - 4x + 3$ for $x \in (-3, 3)$.

3. (20%) Evaluate the following limit.

A. $\lim_{x \rightarrow 0} \left(\frac{1}{x^2}\right)^x$

B. $\lim_{x \rightarrow \pi/2} x \tan(x)$

C. $\lim_{t \rightarrow 1} \frac{t-1}{\ln t}$

D. $\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$

4. (20%) Evaluate the following integrals.

A. $\int_0^{\sqrt[3]{7}} x^2 \sqrt{16 - x^3} dx$

B. $\int x \sin x dx$

C. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

D. $\int \frac{x-2}{x^2+x-2} dx$

5. (10%) Find the tangent line of $x^3 - x^2y + 2xy^3 = 1$ passing through $(1, 1)$.

6. (15%) Find $\iint_R 1 + 2y dA$, where $R = \{(x, y) | 0 \leq 2y \leq x + 2, x \leq y \leq x - 1\}$.

7. (15%) Find the radius of convergence of the power series.

A. $\sum_{n=0}^{\infty} x^n$

B. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+2)!}$

C. $\sum_{n=0}^{\infty} \frac{(4x-5)^{2n+1}}{n^{1/2}}$