# 國立臺北大學109學年度日間學士班暨進修學士班轉學生招生考試試題

系　　別：資訊工程學系日間學士班3年級

科　　目：線性代數



□可 ☑不可使用計算機

1. (10%) Use Gaussian Elimination to solve the following linear system:

$$x_1+x_2+x_3+x_4+x_5=1$$
$$-x_1-x_2+x_5=-1$$
$$-2x_1-2x_2+3x_5=1$$
$$x_3+x_4+3x_5=-1$$
$$x_1+x_2+2x_3+2x_4+4x_5=1$$

2. Let Ax=b be a linear system whose augmented matrix (A|b) has reduced row echelon form $\left[\begin{array}{ccccc|c}1&2&0&3&1&-2\\0&0&1&2&4&5\\0&0&0&0&0&0\\0&0&0&0&0&0\end{array}\right]$

(a) (8%) Find all solutions to the system.

(b)(8%) If $a_1=\begin{bmatrix}1\\1\\3\\4\end{bmatrix}$ and $a_3=\begin{bmatrix}2\\-1\\1\\3\end{bmatrix}$, determine b.

3. (12%) Let $A=\begin{bmatrix}2&1&1\\6&4&5\\4&1&3\end{bmatrix}$. Find elementary matrices $E_1,E_2,E_3$ such that $E_1E_2E_3A=U$ where U is an upper triangular matrix.

4. Let $u_1=[1\ \ 1\ \ 0]^T$, $u_2=[1\ \ 2\ \ 0]^T$, $u_3=[1\ \ 2\ \ 1]^T$, $v_1=[1\ \ 1\ \ 1]^T$, $v_2=[2\ \ 3\ \ 2]^T$, and $v_3=[1\ \ 5\ \ 4]^T$.

(a) (7%) Find the transition matrix from $\{v_1,v_2,v_3\}$ to $\{u_1,u_2,u_3\}$.

(b) (5%) If $x=3v_1+2v_2-v_3$, determine the coordinates of x with respect to $\{u_1,u_2,u_3\}$.

5. (20%) Let ***D*** be the differentiation operator on the set of all polynomials of degree less than 3, and ***D*** is defined as follows:

$$D(p(x))=xp'(x)+p''(x).$$

Now define the matrix **A** representing ***D*** with respect to $[1,x,x^2]$ and the matrix **B** representing ***D*** with respect to $[1,x,1+x^2]$. Show that the matrix **A** and the matrix **B** are similar to each other.

6. (15%) Find the least squares solution of the following system.

$$\begin{cases}-\frac{1}{5}x_1+\frac{1}{5}x_2=2\\ \frac{2}{5}x_1+\frac{1}{5}x_2=1\\ \frac{1}{5}x_1-\frac{2}{5}x_2=4\end{cases}$$

7. (15%) Give the following matrix:

$$\mathbf{A}=\begin{bmatrix}2&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&2\end{bmatrix}.$$

Show that **A** is positive definite based on the ***LU*** factorization of **A**.

試題隨卷繳交