# 國立臺北大學115學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：總體經濟學



☑可 ☐不可使用計算機

**計算與簡答題**

1. Consider a closed economy described by the Solow–Swan growth model with government in discrete time $t$. Aggregate real output ($Y_t$) is allocated among aggregate consumption ($C_t$), gross investment ($I_t$), and government purchases ($G_t$). On the supply side, aggregate output is produced according to the Cobb–Douglas production function $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where $A > 0$, $0 < \alpha < 1$, and $K_t$ and $L_t$ denote aggregate physical capital and labor, respectively. The capital depreciation rate is $d$, and the population growth rate is $n$. The government runs a balanced budget, so that $G_t = \tau Y_t$, where $\tau \in (0,1)$ is the proportional income tax rate. National saving is given by $s(1-\tau)Y_t$, where $s$ is the saving rate and $(1-\tau)Y_t$ denotes aggregate disposable income. When the goods market is in equilibrium, national saving is equal to gross investment. Gross investment is the flow variable that governs the accumulation of physical capital over time.
   (1) Let $k_t = K_t/L_t$ denote capital per worker. Derive the first-order difference equation governing the evolution of capital per worker, that is, $k_{t+1} = g(k_t)$, for this economy. (5%)
   (2) Continuing from part 1, let $y_t$, $c_t$, and $g_t$ denote output per worker, consumption per worker, and government purchases per worker, respectively. In the long run, as $t \to \infty$, derive the steady-state values of capital per worker ($k^*$), output per worker ($y^*$), consumption per worker ($c^*$), and government purchases per worker ($g^*$), expressed in terms of the exogenous parameters. (5%)
   (3) Show mathematically that, under the Golden Rule, the exogenous saving rate must equal one of the other exogenous parameters in this system. Identify this parameter and explain the economic interpretation of the result. (Hint: The Golden Rule maximizes steady-state consumption per worker, which depends on capital accumulation driven by saving out of disposable income.) (10%)

2. Let $Y$ and $r$ denote output and the real interest rate, respectively, in a Keynesian closed economy with the following specifications:
   - Desired consumption: $C^d = 10 + 0.8(Y - T) - 30r$
   - Desired investment: $I^d = 40 - 20r$
   - Government purchases: $G = 30$
   - Taxes: $T = 40$
   - Full-employment ($FE$) output: $\bar{Y} = 182.5$
   - Nominal money supply: $M = 600$
   - Real money demand: $L = 20 + 0.8Y - 200(r + \pi^e)$
   - Expected inflation rate: $\pi^e = 0.1$

(1) Derive the $IS$ equation, the $LM$ equation, and the $AD$ equation for this economy. (5%)

(2) Suppose that the price level is fixed at $P = 5$ in the short run. Compute the short-run equilibrium values of output, the real interest rate, consumption, and investment, and illustrate the short-run equilibrium in an $IS$–$LM$–$FE$ diagram. Then determine the long-run equilibrium values of output, the real interest rate, consumption, investment, and the price level, and illustrate the long-run equilibrium in the same diagram. Explain briefly how the economy moves from the short-run equilibrium to the long-run equilibrium. (10%)

(3) Suppose the government aims to restore the economy to its long-run equilibrium, where $\bar{Y} = 182.5$, while the price level remains fixed at $P = 5$. For each of the following policy scenarios, compute the required policy value **separately** and illustrate your answers in the $IS$–$LM$–$FE$ diagram. (15%)

a. With $M = 600$ and $T = 40$ unchanged, what level of government purchases ($G$) is required?

b. With $M = 600$ and $G = 30$ unchanged, what level of taxes ($T$) is required?

c. With $T = 40$ and $G = 30$ unchanged, what level of nominal money supply ($M$) is required?

3. Consider the following two-period problem. The utility function is

$$u(c_1) + \beta \mathbb{E}[u(c_2)]$$

with $u' > 0, u'' < 0$, where $\beta \in (0,1)$ is the discount factor, $c_t$ is consumption in period $t, \forall t = 1,2$. The budget constraints are

$$c_1 + a_2 = \omega$$

and

$$c_2 = x + a_2(1 + r),$$

where $r$ is the net real interest rate, $a_2$ is net savings at the end of period 1, $\omega$ is a deterministic endowment, and $x$ is a random variable that takes value $\omega + \sigma$ with probability 0.5 and $\omega - \sigma$ with probability 0.5, with $\omega > \sigma \geq 0$.

(1) First consider the case where $\sigma = 0$. Derive the Euler equation (i.e., the optimality condition representing the trade-off between $c_1$ and $c_2$). Explain the economic intuition of this condition in words. (8%)

(2) Assume that the net interest rate $r$ satisfies $\beta(1 + r) = 1$. What is the optimal saving $a_2$? Show your mathematical derivation. (6%)

(3) Suppose that $\beta(1 + r) = 1$ still holds. Now consider a small but positive value for $\sigma$. Under what condition on $u$ do we get positive optimal savings $a_2 > 0$? (6%)

4. Consider the following two-period problem. The representative household utility function is

$$\log(c_1) + \beta \log(c_2),$$

where $\beta \in (0,1)$ is the discount factor, $c_t$ is consumption in period $t, \forall t = 1, 2$.

The budget constraints are

$$c_1 + a_2 = y_1$$

and

$$c_2 = y_2 + a_2(1 + r),$$

where $r$ is the exogenous net real interest rate, $a_2$ is net savings at the end of period 1, and $y_t$ is a deterministic endowment at period $t$, $\forall t = 1, 2$. The household faces a financial constraint:

$$a_2 \geq -\ell,$$

where $\ell \geq 0$ represents the exogenous credit limit.

(1) Define $\lambda$ as the Lagrangian multiplier of the financial constraint. Formulate the Lagrangian equation and derive the intertemporal Euler equation. Express the condition in terms of $c_1, c_2$ and $\lambda$. Please ensure that your representation always implies $\lambda \geq 0$. (8%)

(Hint: You can define other Lagrangian multipliers if necessary)

(2) Using your result from question 1, eliminate $\lambda$ and rewrite the condition as a weak inequality. Why does the constrained household's optimal condition hold with strict inequality? Explain economic intuition in words. (8%)

(Note: You will not receive points if you merely explain the mathematical equation in words)

(3) Given $y_1$ and $\ell$, derive the condition on $y_2$ under which households are unconstrained or constrained. You must specify the threshold of $y_2$ (denoted as $\bar{y}_2$) that separates constrained and unconstrained households as a function of $(\beta, r, y_1, \ell)$ in the condition. Show your mathematical derivation. (8%)

(4) Determine whether the following statement is true or false. If true, please prove it; if false, please disprove or provide a counter-example. (6%)

"If borrowing requires no interest payment (i.e., $r = 0$), the household always borrows to the limit with any combination of $(y_1, y_2, \ell) \subset \mathbb{R}^3_+$."