國立臺北大學 107 學年度日間學士班暨進修學士班轉學生招生考試試題

系（所）組別：通訊工程學系日間學士班 3 年級

電機工程學系日間學士班 3 年級

科　　目：工程數學

☐可 ☑不可使用計算機

1. (10%) Solve the ODE $xy' - y = x^3 e^{y/x}$.
2. It is clear that $y_1(x) = x$ is a solution to the ODE $x(x-1)y'' + xy' - y = 0$.

   (a) (10%) Find another linearly independent solution $y_2(x)$.

   (b) (5%) Compute the Wronskian of $y_1(x)$ and $y_2(x)$.
3. (10%) Solve the ODE $y'' + 9y = 6\sin(3x), y(0) = 1, y'(0) = 2$.
4. (10%) Solve the ODE $y'' + 4y' - 21y = r(t)$ where $r(t) = 1$ for $0 < t \leq 2$ and $r(t) = 0$ for $t > 2$ or $t < 0$, and the initial condition $y(0) = 0, y'(0) = 0$.
5. (15%) Let $f(x)$ be an odd periodic function with period 2π. $f(x) = 0$ for $0 < x < \pi/2$ and $f(x) = x$ for $\pi/2 < x < \pi$. Find the Fourier series representation of $f(x)$.
6. Let the vectors $\boldsymbol{a} = [1,2,4], \boldsymbol{b} = [-5,1,0]$ and $\boldsymbol{c} = [2,-3,3]$. Find

   (a)(4%) $\boldsymbol{a} \cdot \boldsymbol{b}$

   (b)(4%) $\boldsymbol{b} \times \boldsymbol{c}$

   (c)(4%) $\mathbf{c} \cdot (\boldsymbol{b} \times \boldsymbol{a})$
7. (8%) Let $f(x,y,z) = \sqrt{x^2 + 2y^2 + 3z^2}$. Find ∇f and the value of ∇f at the point $(1,-2,3)$.
8. Let the vector function $\boldsymbol{g}(x,y,z) = [3x + 2y, 2yz, x^2 + 2z]$. Find

   (a) (4%) div $\boldsymbol{g}$,

   (b) (4%) curl $\boldsymbol{g}$.
9. (12%) Let $f(x)$ be an even function. $f(x) = 2$ for $0 \leq x < 3$ and $f(x) = 0$ for $x > 3$. Find the Fourier integral representation of $f(x)$.

試題隨卷繳交