

國立臺北大學 104 學年度學士班暨進修學士班轉學生招生考試試題

系 別：資訊工程學士、通訊工程學系學士班 2 年級

考試時間：80 分鐘

科 目：微積分

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可 不可 使用計算機

1.(10%) The region enclosed by the curves $y=x^3$ and $y=\sqrt{x}$ is rotated about $x=-1$. Let S denotes the volume of the resulting solid. Write the integral required to compute S (no need to evaluate)

(a) with y as the variable of the integration, i.e write $S = \int_a^b \dots dy$ (5%)

(b) with x as the variable of the integration, i.e write $S = \int_a^b \dots dx$ (5%)

2.(20%) The Cobb-Douglas Model for the production function is $P = bL^\alpha K^{1-\alpha}$, where b and α are positive constants and $\alpha < 1$. We want to maximize the production P with the constraint that $mL + nK = q$, where m , n and q are positive constants. Show that the maximum production occurs when

$$L = \frac{\alpha q}{m}, \quad K = \frac{(1-\alpha)q}{n}$$

3.(20%) Let $f(x) = x^2 \ln(1+x)$. Compute $f^{(n)}(0)$, the n^{th} derivative of f evaluated at $x=0$, by the following steps.

(a) Find the power series for $f(x)$. (10%)

(b) Use part (a) to derive $f^{(10)}(0)$. (10%)

4. (20%) Please evaluate the integrals:

(a) $\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx$

(b) $\int \frac{\sqrt{x^2-9}}{x^3} dx$

(c) $\int \frac{x^2-x+6}{x^3+3x} dx$

(d) $\int_1^2 (\ln x)^2 dx$

5. (20%) Please sketch the curve $y = \frac{x^3+4}{x^2}$. (You must describe the domain, intercepts, symmetry, asymptotes, interval of increasing or decreasing, local maximum and minimum values, and concavity and points of inflection.)

6. (5%) Prove $\lim_{x \rightarrow 2} \frac{x^2+x-6}{x-2} = 5$ using the ϵ, δ definition of a limit.

7. (5%) Find $\frac{dy}{dx}$ if $y \cos x = 1 + \sin(xy)$.

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