國立臺北大學 113 學年度日間學士班轉學生招生考試試題

學制系級：資訊工程學系日間學士班 3 年級

科　　目：線性代數

☐可　☑不可使用計算機

1. (30%) True or False. In each case, answer **T** if the statement is always true and **F** otherwise.
   (a) If A and B are nonsingular n x n matrices, then $A + B$ is also nonsingular and $(A + B)^{-1} = A^{-1} + B^{-1}$.
   (b) If A and B are n x n matrices, then $(A + B)(A - B) = A^2 - B^2$.
   (c) If $\mathbf{x}$ and $\mathbf{y}$ are two distinct vectors in $\mathbb{R}^n$ and $A\mathbf{x}=A\mathbf{y}$, then $\det(A)=0$.
   (d) If A and B are two equivalent matrices, then their determinants are equal.
   (e) If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ are vectors in a vector space V and $\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k) = \text{Span}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{k-1})$, then $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ are linearly dependent.
   (f) If A is an m x n matrix, then A and A^T have the same nullity.
   (g) Let L: $\mathbb{R}^n \rightarrow \mathbb{R}^n$ be a linear transformation. If $L(\mathbf{x}_1) = L(\mathbf{x}_2)$, then the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ must be equal.
   (h) If an m x n matrix A has linearly dependent columns and $\mathbf{b}$ is a vector in $\mathbb{R}^m$, then $\mathbf{b}$ does not have a unique projection onto the column space of A.
   (i) If Q is an orthogonal matrix, then Q^T also is an orthogonal matrix.
   (j) If A is an n x n matrix, then A and A^T have the same eigenvalues and same eigenvectors.

2. (50%) Given the following linear system:

$$2x_1 - 3x_2 + x_3 = 3$$
$$x_1 - 2x_2 + x_3 = -1$$
$$x_1 - 3x_2 + 2x_3 = 4$$

   (a) Use elementary row operations to convert the above system into strictly triangular system. Then solve the system.
   (b) Solve the linear system using Gaussian-Jordan Reduction.
   (c) Solve the linear system using Cramer's Rule.
   (d) Express the linear system in matrix representation $A\mathbf{x} = \mathbf{b}$, then solve the linear system by computing A^{-1}.
   (e) Compute the rank of A.
   (f) Determine whether the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are linearly independent.
   (g) Determine the basis of the subspace N(A).
   (h) Find the eigenvalues of A.
   (i) For each eigenvalue, find the basis for the corresponding eigenspace.
   (j) Compute A^{20}.

3. (20%) Let $L(\mathbf{x}) = (x_1 + x_2,\ x_2 - x_3)^T$ be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$.
   (a) Determine whether L is a linear transformation.
   (b) Determine the standard matrix representation A and find $L(\mathbf{x})$ for $\mathbf{x} = [1, 2, 3]^T$.

試題隨卷繳交