國立臺北大學 105 學年度日間學士班暨進修學士班轉學生招生考試試題

系　　別：統計學系、經濟學系日間學士班暨進修學士班 2、3 年級

考試時間：80 分鐘

科　　目：微積分

☐可 ☑不可 使用計算機

Part I : Please write the correct answer of each blank in accordance with the question number on the answer sheet. (5 points each blank)

1. (1) Evaluate $\frac{d}{dx}x^x =$ __________.

   (2) Let $f(x)=\begin{cases}\frac{\ln x}{x-1}, x>0, x\neq 1\\ 1, \quad x=1\end{cases}$. Then $f'(1)=$ __________.

   (3) If $f(x)=x^3+2x+1$, then $(f^{-1})'(4)=$ __________.

   (4) If $x>0$, then $\frac{d}{dx}\int_0^{\sqrt{x}} e^{-t^2}dt=$ __________.

   (5) Evaluate $\lim_{x\to 0}\frac{e^x-1-x}{x^2}=$ __________.

   (6) Evaluate $\int_0^{\pi} x\sin x dx=$ __________.

   (7) Evaluate $\int_e^{e^2}\frac{1}{x\ln x}dx=$ __________.

   (8) The radius of convergent of the power series $\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 2^n}$ is __________.

   (9) Let $f(x,y)=xe^y-ye^x$. The gradient of f at $(0,0)$ is $\nabla f(0,0)=$ __________.

   (10) Evaluate $\int_0^4\int_{\sqrt{x}}^2\sqrt{1+y^3}\,dydx=$ __________.

Part II : Give the detailed calculation or proof process of the following questions.

2. Suppose that $\{a_n\}$ is a sequence $a_1=1$ and $a_{n+1}=\sqrt{a_n+2}$, $\forall n\in N$.

   (1) (8%) Prove that $1\le a_n\le 2$, and $a_n\le a_{n+1}$, $\forall n\in N$.

   (2) (6%) Explain why $\{a_n\}$ is convergent.

   (3) (6%) Find $\lim_{n\to\infty}a_n$.

3. (15%) Let $z=f(x,y)$ and $ze^z-x^2y=0$.

   Prove that $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=1-\frac{1}{1+z}$, if $z\neq -1$.

4. (15%) Find all relative maxima, minima, and saddle points of the function :

   $f(x,y)=3x^2y+y^3-3xy$.

試題隨卷繳交