

# 國立臺北大學 109 學年度日間學士班暨進修學士班轉學生招生考試試題

學制系級：經濟學系日間學士班暨進修學士班 2 年級

科目：微積分

第 1 頁 共 1 頁

可 不可使用計算機

Concisely explain your answer with important computation or reasoning steps.

1. Consider the function  $f(x) = [x] - |x|$ , where  $[x]$  denotes the smallest integer that is greater than or equal to  $x$ , and  $|x|$  denotes the greatest integer that is smaller than or equal to  $x$ . (That is,  $[x] = n \in \mathbb{Z}$  such that  $n \geq x > n - 1$ , and  $|x| = m \in \mathbb{Z}$  such that  $m \leq x < m + 1$ .)

1.1 (3%) True or False: The function  $f(x)$  is an increasing function.

1.2 (3%) True or False: The function  $f(x)$  is a continuous function.

1.3 (3%) True or False: The function  $f(x)$  is a concave function.

1.4 (3%) True or False: The function  $f(x)$  is differentiable at  $x = \frac{1}{2}$ .

1.5 (5%) Let  $n$  be a strictly positive integer. Find out  $\int_1^n f(x) dx$ , if exists; if the integral does not exist, explain why.

1.6 (3%) True or False: Given any  $a, b \in \mathbb{R}$  such that  $a < b$ , if there exists a function  $g(x)$  such that

$$\int_a^b f(x) dx = \int_a^b g(x) dx, \text{ then } f(x) = g(x) \text{ for all } x \in [a, b].$$

2. Answer the following questions:

2.1 If  $\int_0^a |x(x-2)| dx = 0$ , then  $a =$  (5%).

2.2  $\int x^5 e^{1-x} dx =$  (5%).

2.3  $\int_{-2}^1 x(x^2-1)^4 dx =$  (5%).

2.4 If  $f(x) = \sin[\sin(\sin x)]$ , then  $\frac{df(x)}{dx} =$  (5%) at  $x = 0$ .

2.5 If  $f(x) = (\ln x)^{1/x}$  for  $x > 0$ , then  $\frac{df(x)}{dx} =$  (5%), and  $\lim_{x \rightarrow \infty} f(x) =$  (5%).

2.6 Given the function  $f(x) = \ln(x-1)$ ,  $x > 1$ , if we use the Taylor polynomial with degree 10 at the point  $x = 2$  to evaluate  $f(3)$ , the approximate value is (5%).

2.7 For the function  $f(x, y) = \sin x + \cos y$ , the Taylor polynomial of degree 2 centered at  $(x, y) = (0, 0)$  is (10%).

2.8 Suppose that the three variables  $x, y$ , and  $z$  satisfy  $x^3 + y^2 + z = 3$  and  $x \cdot y^2 \cdot z^3 = 1$ . Then  $\frac{dx}{dz} =$  (5%) and  $\frac{dy}{dz} =$  (5%) at the point  $(x, y, z) = (1, 1, 1)$ .

3. Given the specified range of the variables, find out the all the critical points of the following functions, and identify the global maxima and minima, if exist:

3.1 (5%)  $f(x) = x - e^x$ , where  $0 \leq x \leq 10$ .

3.2 (10%)  $f(x, y) = x \cdot \sin y$ , where  $x, y \in \mathbb{R}$ .

4. (10%) Consider the infinite series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^a}$ , where  $a > 0$ . Find the necessary and sufficient condition of  $a$  such that the series is convergent, or argue that no such condition exists.