國立臺北大學114學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：個體經濟學

☐可 ☑不可 使用計算機

請清楚標示各小題之題號與答案。未標示答案者不予計分。

1. (7 %)

Consider a variant of the rock-paper-scissors game. The game has two players; each player may simultaneously play one of the following actions: rock, paper, or scissors. The payoff matrix is as follows:

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	$(0,0)$	$(-1,5)$	$(1,-1)$
	Paper	$(5,-1)$	$(0,0)$	$(-1,2)$
	Scissors	$(-1,1)$	$(2,-1)$	$(0,0)$

where (u_1, u_2) denotes Player 1 and Player 2's utilities given the action played by the players, respectively.

a. How many pure strategy Nash equilibria are there?

b. Find the mixed strategy equilibrium.

2. (10 %)

Consider a duopoly market consisting of Firm 1 and Firm 2. Let q_i be Firm i's output, and Firm i's cost function is

$$C_i(q_i) = i \cdot q_i^2.$$

Suppose that the market demand is $P = 100 - Q$, where $Q = q_1 + q_2$.

a. Suppose the two firms compete in Cournot fashion. Find the market equilibrium price and each firm's output.

b. Suppose the two firms form a cartel and evenly share profits. Find the market equilibrium price and each firm's output.

3. (10 %)

Suppose a profit-maximizing firm in a competitive market has the production function $F(L,K) = L + 10K$, where L denotes the labor input, and K denotes the capital input. Let w represents the wage per unit of labor and r represents the rent per unit of capital. Assume the labor and capital markets are competitive.

a. Suppose, in the short run, $K = \bar{K}$. Find the conditional short-run labor demand function $L^*(y, \bar{K})$ and the short-run cost function $C_{SR}(y, \bar{K})$ given the total production y.

b. Suppose, in the long run, all production factors are variable. Find the long-run cost function $C_{LR}(y)$.

4. (23 %)

Consider a competitive labor market with infinitely many workers and employers. There are three types of workers: H, M, and L, where each type's productivity differs. Let π_i denotes type i's productivity, and $\pi_H = 15, \pi_M = 6$, and $\pi_L = 3$. The worker's productivity of each type is public information to everyone in the labor market.

The labor market proceeds as follows.

Stage 1: Hiring. Each employer hires one worker with the job offer of the wage w. Employers cannot distinguish the different types of workers. They only know that $\frac{1}{4}$ of workers are type H, $\frac{1}{2}$ of workers are type M, and $\frac{1}{4}$ of workers are type L. In other words, there is a $\frac{1}{4}$ chance of hiring a worker of Type H, a $\frac{1}{2}$ chance of Type M, and a $\frac{1}{4}$ chance of Type L.

Stage 2: Production. After the workers are hired, the employers then learn the hired worker's type and start production. The employers' profits are $\pi_i - w$, given a type i worker is employed. Assume the reservation wage for all workers is 0, and assume the employers maximize their expected profits.

a. What will the equilibrium wage be in this market?

Suppose the workers can signal their types with an undergraduate degree before entering the labor market. The cost for the workers to obtain an undergraduate degree depends on their type. Let c_i^U denote type i's cost to obtain an undergraduate degree. Employers can observe whether a worker holds an undergraduate diploma but cannot directly observe the worker's type during Stage 1.

b. Consider a partial-separating equilibrium where the employers believe those with an undergraduate degree must be a type H or a type M; otherwise, a type L. Suppose the employers may offer different wages to workers with or without an undergraduate degree. Under this belief, what will the equilibrium wages be in this market? Denote w^U as the wage with an undergraduate degree and w^N as the wage without one.

c. For a type i worker, her utility is

$$U_i(w^U, w^N, c_i^U) = \begin{cases} w^U - c_i^U, \text{if obtaining an undergraduate degree} \\ w^N \quad\quad\quad, \text{otherwise} \end{cases}$$

Consider the following possible options of cost structures (1)—(5).

	c_H^U	c_M^U	c_L^U
(1)	0	1	2
(2)	2	2	5
(3)	2	3	7
(4)	3	5	8
(5)	5	7	9

List all options that can support this equilibrium. In other words, the types of workers that will obtain undergraduate degrees coincide with the employers' beliefs: type H and M will obtain an undergraduate degree, but type L will not.

d. Find the ranges of c_H^U, c_M^U, and c_L^U so that this equilibrium can be supported.

Suppose now that the workers can further signal themselves with a graduate degree. Let c_i^G denote type i's cost to obtain a graduate degree. Employers can also observe whether a worker holds a graduate diploma during Stage 1.

e. Now consider a separating equilibrium where the employers believe those with a graduate degree must be a type H; those with only an undergraduate degree, type M; otherwise, type L. The equilibrium wage for workers with a graduate degree is w^G, for workers with an undergraduate degree, w^U, and for workers with no degree, w^N. For a type i worker, her utility is

$$U_i(w^G, w^U, w^N, c_i^G, c_i^U) = \begin{cases} w^G - c_i^G, \text{if obtaining a graduate degree} \\ w^U - c_i^U, \text{if only obtaining an undergraduate degree} \\ w^N \quad\quad\quad, \text{otherwise} \end{cases}$$

Find the ranges of c_i^U, c_i^G for each type so that this equilibrium can be supported.

5. (25 %)

Ann's utility function is $U(x_1, x_2) = x_1\, x_2^k$, where $k > 0$, x_1 represents the quantity of good 1, and x_2 represents the quantity of good 2. The price of good 1 is 4, and the price of good 2 is 5, respectively. Ann's total income is M, with M>0.

a. Derive the optimal consumption bundle (x_1^*, x_2^*) that maximizes Ann's utility, subject to her budget constraint.

b. Based on your results in (a), compute the income elasticity of demand for good 1.

c. If the price of good 2 increases from 5 to 10, will Ann change her expenditure on the consumption of good 2? Why or why not?

d. If the price of good 1 decreases from 4 to 2, will Ann change her optimal consumption bundle (x_1^*, x_2^*)? Why or why not?

e. Show that Ann's Engel curve is a straight line and derive its slope.

6. (15 %)

Dick is considering purchasing insurance to cover a potential loss. The loss amount is 175, and the probability of the loss occurring is 20%. The insurance provides full compensation, and the premium is 30. Assume Dick's utility function is $U(W) = W^{0.5}$, where W represents his wealth level. Dick's initial wealth is 400.

a. Will Dick choose to purchase the insurance? Calculate the expected utility for both purchasing and not purchasing the insurance, and explain your reasoning.

b. Is Dick risk-averse? Explain your reasoning.

7. (10 %)

Consider a pure exchange economy with two consumers and two goods. Consumer A's utility function is $U_A(x_A, y_A) = x_A\, y_A^2$. Consumer B's utility function is $U_B(x_B, y_B) = x_B^2 y_B$. Consumer A has 2 units of good x. Consumer B has 2 units of good y.

a. Find the Walrasian equilibrium for this exchange economy.

b. Is the Walrasian equilibrium Pareto efficient? Why or why not? Explain your reasoning.

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