

# 國立臺北大學 110 學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科 目：統計學

第1頁 共5頁

☒可 ☐不可使用計算機

## I. 填充題

1. A bivariate probability distribution of X and Y is given by the following table.

|   |   | Table |      |      |
|---|---|-------|------|------|
|   |   | X     |      |      |
|   |   | 20    | 30   | 40   |
| Y | 1 | 0.28  | 0.03 | 0.00 |
|   | 2 | 0.08  | 0.15 | 0.03 |
|   | 3 | 0.04  | 0.06 | 0.06 |
|   | 4 | 0.00  | 0.06 | 0.15 |
|   | 5 | 0.00  | 0.00 | 0.03 |
|   | 6 | 0.00  | 0.00 | 0.03 |

- 1.1 (9分) Calculate the conditional expectations of Y on X = 20, 30, and 40, respectively.
- 1.2 (3分) Calculate the expectation of the conditional expectation of Y on X with respect to X, or  $E_X[E(Y|X)]$ .
2. Suppose that you have a random sample of size  $n > 2$  on a variable y. You are interested in the regression model  $y_i = \beta + u_i$ , where  $\beta$  is an unknown parameter and u follows the normal distribution with zero mean and finite variance  $\sigma^2$ . Under the Gauss-Markov assumptions, answer the following questions.
- 2.1 (3分) What is the least squares estimator of  $\beta$ ?
- 2.2 (3分) What is the unbiased estimator of  $\sigma^2$ ? Suppose you choose an estimator  $\hat{\beta}^* = \frac{1}{2} (y_1 + y_n)$ .
- 2.3 (3分) What is the expectation of this estimator ( $\hat{\beta}^*$ )?
- 2.4 (3分) What is the variance of this estimator ( $\hat{\beta}^*$ )?
- 2.5 (3分，第2.3與2.4小題皆需答對則此題答對才可得分) Is this estimator ( $\hat{\beta}^*$ ) a best linear unbiased estimator?
- 2.6 (3分，第2.3與2.4小題皆需答對則此題答對才可得分) Is this estimator ( $\hat{\beta}^*$ ) consistent?
3. The following least squares estimates have been obtained for a Cobb-Douglas production function  $y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \hat{u}_i$ , where y is the log of output,  $x_1$  is the log of labor input, and  $x_2$  is the log of capital input. All data have been mean centered (zero mean). Also, the sample size is  $n = 22$  and all of the estimates are given as follows:  $\hat{\beta}_1 = 0.5$ ,  $\hat{\beta}_2 = 0.4$ ,  $\widehat{\text{Var}}(\hat{\beta}_1) = 0.05$ ,  $\widehat{\text{Var}}(\hat{\beta}_2) = 0.02$ ,  $\widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_2) = 0.01$ .
- 3.1 (3分) Calculate a 90% confidence interval for  $\tau = \beta_1 + \beta_2$ . Here the t value is  $t_{(\alpha/2=0.05, df=20)} = 1.725$ .
- 3.2 (2分，第3.1小題需答對則此題答對才可得分) Do you conclude that the production function is constant return to scale?

## II. 選擇題(每小題5分)

1. Three regression models are estimated as follows (standard errors in parentheses).

$$\hat{y} = -391.55 + 14.20x_1 + 15.74x_2 - 81.83x_3 \quad (\text{i})$$

(142.65) (1.94) (3.97) (29.15)

$$\hat{y} = -425.00 + 13.95x_1 + 16.72x_2 - 76.81x_3 \quad (\text{ii})$$

(121.44) (1.48) (3.02) (21.85)

$$\hat{y} = -391.55 + 14.20x_1 + 15.74x_2 - 81.83x_3 \quad (\text{iii})$$

(139.78) (1.80) (3.76) (27.13)

Equations (i), (ii), and (iii) are estimated by the following three methods, respectively. It is noted that OLS is Ordinary Least Squares, FGLS is Feasible Generalized Least Squares, and HC standard errors is Heteroscedasticity Consistent standard errors.

- (A) FGLS, OLS, and OLS with HC standard errors  
(B) OLS, OLS with HC standard errors, and FGLS  
(C) FGLS, OLS with HC standard errors, and OLS  
(D) OLS with HC standard errors, FGLS, and OLS  
(E) OLS with HC standard errors, OLS, and FGLS

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2. Consider the following estimated regression model  $\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 x$ , where  $\hat{\beta}_1 = 5.83$ ,  $\hat{\beta}_2 = 0.869$ ,  $\widehat{\text{Var}}(\hat{\beta}_1) = 1.513$ ,  $\widehat{\text{Var}}(\hat{\beta}_2) = 0.014$ . If all values of  $y$  are divided by 10, what will  $\hat{\beta}_1$  and  $\widehat{\text{Var}}(\hat{\beta}_2)$  result?
- (A) 5.83 and 0.14  
(B) 0.583 and 0.0014  
(C) 5.83 and 0.014  
(D) 58.3 and 0.14  
(E) 0.583 and 0.014
3. We examine the wage difference among four groups: married men, married women, single men, and single women. The regression model with dummy variables defined for these four groups, in which single men are chosen as the base group, is given by  $y = \beta_1 + \beta_2 x + \beta_3 D_1 + \beta_4 D_2 + \beta_5 D_3 + u$ , where  $y$  is wage,  $x$  is schooling year,  $D_1$  is married men,  $D_2$  is married women,  $D_3$  is single women, and  $\beta_3 > \beta_4 > \beta_5 > 0$ . Suppose now married women are chosen as the base group, what are the expected signs of  $\beta_3$ ,  $\beta_5$ , and  $\alpha$ , where  $\alpha$  is defined with respect to the dummy for single men.
- (A)  $\beta_3 > 0$ ,  $\beta_5 > 0$ , and  $\alpha > 0$   
(B)  $\beta_3 > 0$ ,  $\beta_5 < 0$ , and  $\alpha > 0$   
(C)  $\beta_3 > 0$ ,  $\beta_5 < 0$ , and  $\alpha < 0$   
(D)  $\beta_3 > 0$ ,  $\beta_5 > 0$ , and  $\alpha < 0$   
(E)  $\beta_3 < 0$ ,  $\beta_5 < 0$ , and  $\alpha < 0$

### III. 選擇題(每小題3分)

1. If  $X$  is a random variable such that  $E(X) = 3$  and  $E(X^2) = 13$ , which is a lower bound for the probability  $P(-2 < X < 8)$  by Chebyshev's inequality.
- (A) 0.16  
(B) 0.72  
(C) 0.75  
(D) 0.84
2. A measure of skewness is defined by  $S = \frac{E[(X - \mu_X)^3]}{(\sigma^2)^{3/2}}$ . Where  $\mu_X = E(X)$ . If a researcher find  $S = -2$  for a special distribution, How about the skewness of this distribution?
- (A) skew to the right.  
(B) skew to the left.  
(C) symmetric.  
(D) this distribution is skew, but we cannot judge by measure of skewness directly.
3. Let  $f(x, y) = \frac{1}{40}$ ,  $0 \leq x \leq 10$ ,  $10 - x \leq y \leq 14 - x$ , be the joint p.d.f. of  $X$  and  $Y$ . What is the marginal p.d.f. of  $X$ ?
- (A)  $1/2$   
(B)  $1/5$   
(C)  $1/10$   
(D)  $1/15$
4. There are times when a shifted exponential model is appropriate. That is, let the p.d.f. of  $X$  be  $f(X) = \frac{1}{\theta} e^{-(X-\delta)/\theta}$ ,  $\delta \leq X < \infty$ .
- What is the variance of  $X$ ?
- (A)  $\theta + \delta$   
(B)  $\theta^2$   
(C)  $(\theta + \delta)^2$   
(D)  $\theta$

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5. Bowl  $B_1$  contains two red and five white chips, and bowl  $B_2$  contains four red and three chips. A fair die is cast. If the output is a multiple of 3 (namely, 3 or 6), a chip is taken from bowl  $B_2$ ; Otherwise a chip is taken from bowl  $B_1$ . Given that the selected chip is red, what is the conditional probability that it was taken from bowl  $B_1$ .

(A)  $\frac{8}{21}$

(B)  $\frac{1}{3}$

(C)  $\frac{1}{2}$

(D)  $\frac{10}{21}$

6. If  $X$  is normally distributed with a mean of 6 and a variance of 25, What is  $P(|X - 6| < 15)$ ?

(A) 0.9974

(B) 0.9876

(C) 0.4514

(D) 0.8664

7. Let the joint p.d.f. of  $X$  and  $Y$  be defined by  $f(x, y) = \frac{x+y}{32}$ ,  $x = 1, 2$ ,  $y = 1, 2, 3, 4$ . What is  $P(Y = 2X)$ ?

(A)  $\frac{9}{32}$

(B)  $\frac{3}{32}$

(C)  $\frac{3}{16}$

(D)  $\frac{1}{4}$

8. Follow the question 7. What is the correlation coefficient  $\rho$  between  $X$  and  $Y$ ?

(A) -0.5

(B) -0.037

(C) -0.067

(D) -0.027

9. Let  $\bar{X}$  be the mean of a random sample of size 36 from an exponential distribution with mean 3. What is the approximation of  $P(2.5 \leq \bar{X} \leq 4)$  by central limit theorem?

(A) 0.7932

(B) 0.7102

(C) 0.5483

(D) 0.6853

10. Let  $Z_1, Z_2, \dots, Z_7$  be a random sample of size  $n = 7$  from the standard normal distribution  $N(0, 1)$ . Let  $W = Z_1^2 + Z_2^2 + \dots + Z_7^2$ . What is  $P(1.69 < W < 14.07)$ ?

(A) 0.99

(B) 0.875

(C) 0.95

(D) 0.925

## IV. 計算題

- (10 分) If the moment generating function of  $X$  is  $M(t) = 0.45 + 0.55e^t$ , what is  $E(X)$  and variance of  $X$ ?
- (10分) Let the p.d.f. of  $X$  be defined by  $f(x) = x^3/4$ ,  $0 < x < 2$ . Find the p.d.f. of  $Y = X^2$ .

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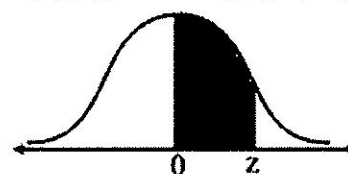
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☒可 ☐不可使用計算機

| 標準常態分配表 |        |        |        |        |        |        |        |        |        |        |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| z       | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
| 0.0     | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1     | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2     | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3     | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4     | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5     | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6     | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7     | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8     | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9     | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0     | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1     | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2     | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3     | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4     | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5     | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6     | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7     | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8     | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9     | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0     | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1     | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2     | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3     | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4     | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5     | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6     | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7     | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8     | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9     | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0     | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1     | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2     | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3     | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4     | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5     | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |



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The Chi-Square distribution table

| df  | 0.995  | 0.99   | 0.975  | 0.95   | 0.9    | 0.1     | 0.05    | 0.025   | 0.01    | 0.005   |
|-----|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| 1   | ---    | ---    | 0.001  | 0.004  | 0.016  | 2.706   | 3.841   | 5.024   | 6.635   | 7.879   |
| 2   | 0.01   | 0.02   | 0.051  | 0.103  | 0.211  | 4.605   | 5.991   | 7.378   | 9.21    | 10.597  |
| 3   | 0.072  | 0.115  | 0.216  | 0.352  | 0.584  | 6.251   | 7.815   | 9.348   | 11.345  | 12.838  |
| 4   | 0.207  | 0.297  | 0.484  | 0.711  | 1.064  | 7.779   | 9.488   | 11.143  | 13.277  | 14.86   |
| 5   | 0.412  | 0.554  | 0.831  | 1.145  | 1.61   | 9.236   | 11.07   | 12.833  | 15.086  | 16.75   |
| 6   | 0.676  | 0.872  | 1.237  | 1.635  | 2.204  | 10.645  | 12.592  | 14.449  | 16.812  | 18.548  |
| 7   | 0.989  | 1.239  | 1.69   | 2.167  | 2.833  | 12.017  | 14.07   | 16.013  | 18.475  | 20.278  |
| 8   | 1.344  | 1.646  | 2.18   | 2.733  | 3.49   | 13.362  | 15.507  | 17.535  | 20.09   | 21.955  |
| 9   | 1.735  | 2.088  | 2.7    | 3.325  | 4.168  | 14.684  | 16.919  | 19.023  | 21.666  | 23.589  |
| 10  | 2.156  | 2.558  | 3.247  | 3.94   | 4.865  | 15.987  | 18.307  | 20.483  | 23.209  | 25.188  |
| 11  | 2.603  | 3.053  | 3.816  | 4.575  | 5.578  | 17.275  | 19.675  | 21.92   | 24.725  | 26.757  |
| 12  | 3.074  | 3.571  | 4.404  | 5.226  | 6.304  | 18.549  | 21.026  | 23.337  | 26.217  | 28.3    |
| 13  | 3.565  | 4.107  | 5.009  | 5.892  | 7.042  | 19.812  | 22.362  | 24.736  | 27.688  | 29.819  |
| 14  | 4.075  | 4.66   | 5.629  | 6.571  | 7.79   | 21.064  | 23.685  | 26.119  | 29.141  | 31.319  |
| 15  | 4.601  | 5.229  | 6.262  | 7.261  | 8.547  | 22.307  | 24.996  | 27.488  | 30.578  | 32.801  |
| 16  | 5.142  | 5.812  | 6.908  | 7.962  | 9.312  | 23.542  | 26.296  | 28.845  | 32      | 34.267  |
| 17  | 5.697  | 6.408  | 7.564  | 8.672  | 10.085 | 24.769  | 27.587  | 30.191  | 33.409  | 35.718  |
| 18  | 6.265  | 7.015  | 8.231  | 9.39   | 10.865 | 25.989  | 28.869  | 31.526  | 34.805  | 37.156  |
| 19  | 6.844  | 7.633  | 8.907  | 10.117 | 11.651 | 27.204  | 30.144  | 32.852  | 36.191  | 38.582  |
| 20  | 7.434  | 8.26   | 9.591  | 10.851 | 12.443 | 28.412  | 31.41   | 34.17   | 37.566  | 39.997  |
| 21  | 8.034  | 8.897  | 10.283 | 11.591 | 13.24  | 29.615  | 32.671  | 35.479  | 38.932  | 41.401  |
| 22  | 8.643  | 9.542  | 10.982 | 12.338 | 14.041 | 30.813  | 33.924  | 36.781  | 40.289  | 42.796  |
| 23  | 9.26   | 10.196 | 11.689 | 13.091 | 14.848 | 32.007  | 35.172  | 38.076  | 41.638  | 44.181  |
| 24  | 9.886  | 10.856 | 12.401 | 13.848 | 15.659 | 33.196  | 36.415  | 39.364  | 42.98   | 45.559  |
| 25  | 10.52  | 11.524 | 13.12  | 14.611 | 16.473 | 34.382  | 37.652  | 40.646  | 44.314  | 46.928  |
| 26  | 11.16  | 12.198 | 13.844 | 15.379 | 17.292 | 35.563  | 38.885  | 41.923  | 45.642  | 48.29   |
| 27  | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741  | 40.113  | 43.195  | 46.963  | 49.645  |
| 28  | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916  | 41.337  | 44.461  | 48.278  | 50.993  |
| 29  | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087  | 42.557  | 45.722  | 49.588  | 52.336  |
| 30  | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256  | 43.773  | 46.979  | 50.892  | 53.672  |
| 40  | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805  | 55.758  | 59.342  | 63.691  | 66.766  |
| 50  | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167  | 67.505  | 71.42   | 76.154  | 79.49   |
| 60  | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397  | 79.082  | 83.298  | 88.379  | 91.952  |
| 70  | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527  | 90.531  | 95.023  | 100.425 | 104.215 |
| 80  | 51.172 | 53.54  | 57.153 | 60.391 | 64.278 | 96.578  | 101.879 | 106.629 | 112.329 | 116.321 |
| 90  | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

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