國立臺北大學115學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：個體經濟學

☑可 ☐不可 使用計算機

※請清楚標示各小題之題號與答案，未標示題號者不予計分。

1. (17 %)

Consider a utility function $U: \mathbb{R}_+^2 \to \mathbb{R}$, $U(x_1, x_2) = |x_1 - x_2|$.

This utility function represents a preference relation $\succsim$. Equivalently, for any bundles $(x_1, x_2), (y_1, y_2)$,

$$(x_1, x_2) \succsim (y_1, y_2) \text{ if and only if } U(x_1, x_2) \geq U(y_1, y_2).$$

Consider the following properties.

> **Definition.** A preference relation $\succsim$ is *convex* if for any $\alpha \in (0,1)$
> $$(x_1, x_2) \succsim (z_1, z_2) \text{ and } (y_1, y_2) \succsim (z_1, z_2)$$
> $$\Rightarrow (\alpha x_1 + (1-\alpha) y_1 , \alpha x_2 + (1-\alpha) y_2) \succsim (z_1, z_2).$$

> **Definition.** A preference relation $\succsim$ is *homothetic* if for any $t > 0$
> $$(x_1, x_2) \succsim (y_1, y_2)$$
> $$\Rightarrow (tx_1, tx_2) \succsim (ty_1, ty_2).$$

(a) True/False/Uncertain: Is $\succsim$ transitive?

(b) True/False/Uncertain: Is $\succsim$ convex?

(c) True/False/Uncertain: Is $\succsim$ homothetic?

Let p_1, p_2 be the prices of the first and the second good, respectively, and let m be the total income that can be spent.

(d) Find the demand functions of two goods $x_1(p_1, p_2, m), x_2(p_1, p_2, m)$ under this preference.

(e) Let $p_2 = 1$. Suppose p_1 changes from $1/2$ to 2. Find the equivalent variation and the compensating variation.

2. (9 %)

Consider two players in a game: Legislator (L) and the Judiciary (J). The Legislator and the Judiciary negotiate a bill B to be executed from a policy space $\mathcal{B} = [0,10]$, where $B_0 = 0$ is the status quo. Legislator's favorite policy is $B_L^* = 9$, and Judiciary's favorite policy is $B_J^* = 3$.

The game has 2 stages and proceeds as follows:

> **Stage 1**: The Legislator chooses a bill $B_L \in [0,10]$.
>
> **Stage 2**: After seeing B_L, the Judiciary can choose to *pass* or *reject* the bill. If the bill is passed, the executed bill will be $B = B_L$. If the bill is rejected, the society goes back to the status quo, $B = B_0 = 0$.

Define player i's payoff as $\pi_i = |B_i^* - B|$. Assume that the Judiciary will not reject the bill if rejecting the bill is indifferent to her.

(a) Under the Subgame Perfect Nash Equilibrium, what is the equilibrium bill passed, B_{SPNE}?

(b) Fix $B_L^* = 9$, and make B_J^* variable. Express the passed bill under the SPNE as the function of the Judiciary's preference $B(B_J^*)$. Make sure you have discussed all possibilities $B_J^* \in [0,10]$.

3. (24 %)

Consider a duopoly market that consists of two firms, OpenAI and Google. The competition proceeds as follows:

> **Stage 1**: Each firm simultaneously determines its own R&D input level, x_i.
>
> **Stage 2**: Knowing the R&D input level of each other, each firm simultaneously determines its own output level, q_i.

Each firm has the following cost function:

$$C_i(q_i, x_i, x_j) = (70 - (x_i + x_j))q_i + 2x_i^2,$$

where x_j is the R&D level the other firm chooses. Let the inverse demand function of the market be $P = 100 - \frac{1}{3}Q$, where $Q = q_O + q_G$.

(a) Consider the Cournot duopoly equilibrium. Given x_O, x_G determined, find the equilibrium output chosen by each firm in Stage 2.

(b) Following (a), express the profit functions as functions of x_O, x_G.

(c) Following (b), find each firm's best response function of R&D input level in Stage 1.

(d) Following (c), find each firm's equilibrium level of R&D input.

(e) Find the social welfare under this scenario, where the social welfare is defined as the sum of the consumer surplus and the firms' profits.

Now, suppose the two firms agree to collude in Stage 1 but continue to compete in a Cournot manner in Stage 2. That is, the two firms will collude with the R&D input level in Stage 1, and then individually decide the output levels in Stage 2. The two firms agree to coordinate R&D levels to maximize total joint profit.

(f) Find the optimal level of each firm's R&D input under collusion. Compare your answer with (d). Intuitively explain the difference.

(g) Find the social welfare under this scenario.

4. (20 %)

Consider a firm whose CEO's compensation equals a fraction λ of the firm's operating profit. The CEO is risk-averse and has utility U(W)=ln (W), where W is the compensation. The firm's production function is $y= \theta \sqrt{k}$, where $k \geq 0$ is the capital input and θ is a random productivity shock with Probability(θ=2)=1/2, Probability(θ=1)=1/2.

(a) The output price is p and the unit cost of capital is r>0. Please derive the CEO's optimal choice of capital k. (10%)

(b) Compare the optimal capital input under uncertainty with the risk-neutral benchmark case. Is the optimal capital choice under uncertainty higher or lower than in the deterministic case? Explain your result using both mathematical reasoning and economic intuition. (10 %)

5. (15 %)

Consider a product market in which a private firm faces an inverse demand function $p(Q) = a - b\,Q$ and private cost $\frac{c}{2}Q^2$. The environmental damage $D(Q) = \frac{d}{2}\,Q^2$, where a, b, c, d > 0.

Assume that each unit of output produces one unit of carbon emissions. Firms engage in Environmental Corporate Social Responsibility (ECSR) by internalizing a fraction $\gamma \in [0,1]$ of units of carbon emissions. The government can impose a per-unit carbon tax t on output. Derive the equilibrium price and how an increase in the degree of ECSR γ affects the equilibrium price.

6. (15 %)

Consider a two-good (X,Y), two-factor (L,K) general equilibrium economy. Production technologies are $X = F^X(L_X, K_X)$ and $Y = F^Y(L_Y, K_Y)$, where both F^X and F^Y exhibit constant returns to scale and are strictly concave. Factor endowments satisfy $L_X + L_Y = \bar{L}$ and $K_X + K_Y = \bar{K}$. Consumer preferences are represented by standard Cobb-Douglas functions.

(a) Show that any competitive equilibrium allocation must satisfy production efficiency. (7 %)

(b) Now suppose production of good X involves a factor-use externality (e.g., the output of X depends on an aggregate factor use term in addition to the firm's own inputs). Explain how the competitive equilibrium deviates from efficiency. (8 %)

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