

國立臺北大學 112 學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科 目：個體經濟學

第1頁 共2頁

☐可 ☒不可 使用計算機

1. Mary buys commodity x and y that maximize her utility $U(x, y) = \alpha x + y$ with $\alpha > 0$. Let her income be I , the price of x be p_x , and the price of y be p_y . Answer the following questions in terms of α, p_x, p_y and I . (12 %)
- (a) (2%) Explain why x is an economic good for Mary.
- (b) (6%) Derive and graph Mary's demand curve for x .
- (c) (4%) Suppose that one optimal solution x^* is nonzero and independent of p_y . Find Mary's price elasticity and income elasticity of demand for x under x^* .

2. A utility-maximizing consumer chooses between commodity x and y with the utility function $U(x, y) = \sqrt[3]{x + xy}$ while receiving a total income of 19. Let p_x denote the price of x , and suppose that the price of y is 1. Answer the following questions. (14 %)

- (a) (6%) Find the optimal solutions for x and y when $p_x = 0.4$ and 0.2.

Now suppose that p_x decreases from 0.4 to 0.2.

- (b) (4%) What is the substitution effect of this reduction in price?
- (c) (4%) What is the compensating variation of this reduction in price?

3. Pricing Strategy of a Monopoly (26 %)

Planet Gym is the only gym in town. Providing a session of workout to a customer is at a marginal cost of c . This marginal cost is perfect information. The gym is large enough so that even in peak hours there is no externality between the customers.¹

Planet Gym has two forms of feasible pricing strategies. It can set a uniform price per session for all customers with free entrance, or it can set a two-part tariff with a membership fee and a price per session conditional on paying the membership fee.

The market demand of gym session at price p per session is $q = \Delta - \delta \cdot p$. The market demand is perfect information. To guarantee existence of an equilibrium, assume that $\Delta > \delta \cdot c$.

3-1. Uniform Price per Session (10 %)

Suppose in this part of the question that the Planet Gym takes a simple pricing strategy: to set a uniform price per session. Given this form of pricing strategy, it maximizes its profit. Please find Planet Gym's supply of gym sessions. Please accompany your answer with a consistent figure (graph).

3-2. Two-Part Tariff (10 %)

Suppose in this part of the question that the Planet Gym adopts a two-part tariff: to set a membership fee and a price per session for customers who have paid the membership fee. The membership fee is required for entrance. Given this form of pricing strategy, the Planet Gym maximizes its profit. Please find the optimal membership fee, the optimal price per session, and the equilibrium gym sessions traded. Please accompany the above answers with a consistent figure (graph).

3-3. The Optimal Pricing Strategy (6 %)

From what you have found in 3-1 and 3-2, what is the optimal pricing strategy for the Planet Gym? Is the optimal pricing strategy efficient from the social perspective? Is the optimal pricing strategy also the best for the customers? Briefly explain.

¹ This assumption might not be realistic, but it allows us to focus on what we are really interested to analyze.

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第2頁 共2頁

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4. A cost-minimizing firm has the production function $Q(L, K) = KL + K + L$. Suppose that the price of labor services (L) is 1, and let r denote the price of capital services (K). If the firm aims to produce 20 units of output, for what values of r would the firm use (9%)
- (a) (3%) only labor?
 - (b) (3%) only capital?
 - (c) (3%) both labor and capital?
5. A price-taking firm in a perfectly competitive market uses three factors of production: labor (L), capital (K), and material (M). The firm's faces the production function $Q(L, K, M) = L^\alpha K^\beta M^\gamma$ with $\alpha, \beta, \gamma > 0$. (15%)
- (a) (3%) For what condition (in terms of α, β, γ) would this production function exhibit constant returns to scale?
 - (b) (6%) Show that the total value created by the firm under constant returns to scale is the exact sum of the values created by each factor's marginal productivity.
 - (c) (6%) Explain the economic intuition behind the previous result.

6. Human Right Protection (24%)

Human right protection in Taiwan is enforced by the judicial system (e.g. the Constitution and the Constitutional Court). This is due to the non-exclusive and non-rival properties of the public goods. Let's take a game theoretic approach to analyze the provision of human right protection

There are $n \geq 2$ citizens in the society. Each of them, indexed by $i = 1, 2, \dots, n$, can freely decide to provide human right protection of level r_i as any real number within zero and one (that is, $0 \leq r_i \leq 1$ from each citizen i). Human right protection of level r_i is at a linear cost of $c(r_i) = c \cdot r_i$ to the provider, with $c > 0$. If multiple people provide different levels of human right protection, the actual level everyone in the society enjoys is the maximum among them. Denote this actual level of human right protection in the society as r , and $r = \max\{r_i\}$ (that is, r equals to the highest level of human right protection among all providers).²

Human right protection is a public good in the sense that if level r is provided, every citizen enjoys a positive utility from it. Among the citizens, a proportion $m > 0$ of them highly value human right protection, and they earn utility $u_h \cdot r$ from human right protection of level r . Among the citizens, a proportion $1 - m > 0$ of them slightly value human right protection, and they earn utility $u_l \cdot r$ from human right protection of level r . Suppose that $u_h > c > u_l > 0$ and $n \cdot u_l > c$. The utility of human right and the cost of provision for each citizen are perfect information. There is no other externality.

Please find the Nash equilibrium level of human right protection by explaining the following:

- (a) Show free-riding incentive: if some citizens provide human right protection, each citizen except the one providing the maximum level of human right protection has incentive to unilaterally deviate to provide zero human right protection, regardless of her value on human right protection.
- (b) Show that if no one provides human right protection, one citizen among those who highly value human right has incentive to unilaterally deviate from providing zero human right protection.

Please characterize the above Nash equilibrium. Specifically, how many citizens provide human right protection? Who is(are) the provider(s)? In equilibrium, what is the actual level of human right protection in the society?

² For example, if Citizen 1 provides $r_1 = 0.1$ and Citizen 2 provides $r_2 = 0.7$, and no one else provides positive level of human right protection, the actual level of human right protection in the society is $r = \max\{r_1, r_2\} = 0.7$.