

# 國立臺北大學 108 學年度日間學士班轉學生招生考試試題

系 別：資訊工程學系日間學士班 3 年級

科 目：線性代數

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可 不可使用計算機

1. (10%) Find the reduced row echelon form of the matrix  $\begin{bmatrix} -1 & 1 & -1 & 3 \\ 3 & 1 & -1 & -1 \\ 2 & -1 & -2 & -1 \end{bmatrix}$

2. (10%) Use the elementary row operations to find the inverse of the matrix  $\begin{bmatrix} 2 & 0 & 5 \\ 0 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

3. (10%) Let  $A=[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$  be a  $3 \times 3$  matrix and suppose that  $2\mathbf{a}_1 + \mathbf{a}_2 - 4\mathbf{a}_3 = \mathbf{0}$ . How many solutions will the system  $A\mathbf{x} = \mathbf{0}$  have?

4. (10%) Let  $x_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$ ,  $x_2 = \begin{bmatrix} -2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ , and  $x_3 = \begin{bmatrix} 1 \\ 0 \\ 7 \\ 7 \end{bmatrix}$ . Determine whether the vectors are linearly independent.

5. (10%) Show that  $e^x, e^{-x}, e^{2x}$  are linearly independent in  $C[0,1]$  where  $C[0,1]$  is the set of continuous functions defined on  $[0,1]$ .

6. Give  $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 5 & 2 \\ 0 & 0 \end{bmatrix}$ .

(a)(6%) Find the bases for  $N(\mathbf{A}^T)$ , namely, the null space of  $\mathbf{A}^T$ .

(b)(6%) Find the bases for  $R(\mathbf{A})$ , namely, the column space of  $\mathbf{A}$ .

(c)(6%) Show that  $N(\mathbf{A}^T)$  and  $R(\mathbf{A})$  are orthogonal complement.

7. Define a mapping from  $\mathbf{R}^2$  to  $\mathbf{R}^2$ :  $L(\mathbf{x}) = (x_1 + 3x_2 + 2, 5x_1 + x_2)^T$  for each  $\mathbf{x} \in \mathbf{R}^2$ .

(a)(6%) Determine whether the mapping  $L$  is a linear transformation.

(b)(6%) Find a matrix  $\mathbf{A}$  to perform the mapping  $L(\mathbf{x})$ .

8. Give  $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(a)(6%) Find the eigenvalues and the corresponding eigenvectors for  $\mathbf{A}$ .

(b)(6%) Diagonalize  $\mathbf{A}$  as  $\mathbf{A} = \mathbf{X}\mathbf{D}\mathbf{X}^{-1}$  where  $\mathbf{D}$  is a diagonal matrix.

9. (8%) Prove that if  $\mathbf{A}$  is an  $n \times n$  orthogonal matrix then  $\mathbf{a}_1\mathbf{a}_1^T + \mathbf{a}_2\mathbf{a}_2^T + \cdots + \mathbf{a}_n\mathbf{a}_n^T = \mathbf{I}$ , where  $\mathbf{a}_i$  is the  $i$ th column of  $\mathbf{A}$ , and  $\mathbf{I}$  is the identity matrix.