國立臺北大學 111 學年度日間學士班轉學生招生考試試題

學制系級：統計學系日間學士班 3 年級

科　　目：機率概論

☑可　☐不可 使用計算機

1. (10%) Let C_1, C_2 and C_3 be mutually independent events, where $P[C_1] = \frac{2}{6}$, $P[C_2] = P[C_3] = \frac{1}{6}$. Find $P[C_1 \cup C_2 \cup C_3]$ and $P[(C_1^C \cap C_2^C) \cup C_3]$, where C^C is the complement of C.

2. (20%) An urn contains five balls, one marked W and four marked L. You and another player take turns selecting a ball at random from the urn, one at a time. The first person to select W ball is the winner.
   (a) (5%) Write down the sample space if balls are selected without replacement.
   (b) (5%) Continued (a), find the probability that you are the winner at third draw.
   (c) (5%) If balls are selected with replacement. Let X denote the number of selected balls if you are the winner. Find the probability mass function.
   (d) (5%) Derive $E[X]$. [A detailed derivation is required.]

3. (10%) Let X have the probability mass function

$$f_X(x) = P[X = x] = \frac{3}{2}\left(\frac{1}{2}\right)^{2|x|}, x = \pm 1, \pm 2, \ldots$$

   (a) (5%) Find $P[X = 1,2,3,\cdots]$.
   (b) (5%) Find $E[X]$ and $Var(X)$. [A detailed derivation is required and the answer has to be simplified.]

4. (10%) Suppose that 500 points are selected independently and at random from the unit square

$$\{(x, y): 0 \le x < 1, 0 \le y < 1\}.$$

   Let W equal the number of points that fall into $\{(x, y): x^2 + y^2 < 1\}$.
   (a) (5%) Write down the distribution of W and the corresponding probability mass function.
   (b) (5%) Find the mean and variance of W. [A detailed derivation is required.]

5. (15%) Red Rose tea randomly began placing 1 of 6 English porcelain miniature figurines in a 100-bag box of the tea, selecting from 6 nautical figurines, which are numbered 1 to 6.
   (a) Let X denote the number of Red Rose tea that a family would have to buy to have at least three of the nautical figurine having number 1. Write down the probability mass function of X.
   (b) Derive the moment generating function of X. [A detailed derivation is required.]
   (c) On the average, how many boxes of tea must be purchased by a customer to obtain a complete collection consisting of the 6 nautical figurines.

6. (10%) Let $f(x) = \frac{1}{8}, 0 \le y \le 4, y \le x \le y + 2$ be the joint pdf of X and Y.
   (a) Find the conditional pdf of X given $Y = y$ and of Y given $X = x$.
   (b) Find the conditional expectation of X given $Y = y$.

7. (5%) Let X_1 and X_2 have the joint probability density function

$$f(x_1, x_2) = \frac{1}{2}, 0 < x_1 < x_2 < 2$$

Let $Y = \frac{X_1}{X_2}$. Find the probability density of Y.

8. (10%) Two components operate in parallel in a device. The device fails when and only when both components fail. The lifetimes, X_1 and X_2, of the respective components are independent and identically distributed with an exponential distribution with the probability density function.

The cost of operating the device is $Z = Y_1 + 2Y_2$, where $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$

(a) Find the joint probability density function of Y_1 and Y_2.

(b) Find E[Z].

9. (10%) In a college health fitness program, let X denote the weight in kilograms of a male freshman at the beginning of the program and Y denote the weight change during a semester. Assume X and Y have a bivariate normal distribution with $\mu_X = 75$ and $\mu_Y = 5$, $\sigma_X^2 = 64$, $\sigma_Y^2 = 2.56$ and $\rho = -0.6$. The joint probability density function of a bivariate normal distribution is defined as

$$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left(-\frac{q(x,y)}{2}\right)$$

where $q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{x-\mu_Y}{\sigma_Y}\right)^2\right]$.

(a) Derive the marginal pdf of Y and find $P[2.5 \leq Y \leq 6.5]$. [A detailed derivation is required.]

(b) Derive the conditional pdf of Y given $X = x$ and find $P[2.5 \leq Y \leq 6.5 | X = 80]$. [A detailed derivation is required.]

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Table 1: Area under the standard normal curve

z	P[Z<z]								
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990

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