

國立臺北大學 106 學年度學士班暨進修學士班轉學生招生考試試題

系別：統計學系 2、3 年級年級(學士班)

科目：微積分

考試時間：80 分鐘

第 1 頁，共 1 頁

可 不可使用計算機

1. Let  $f(x) = \begin{cases} x^2 \ln|x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$ 
  - (a) (7%) Find  $\lim_{x \rightarrow 0} f(x)$ .
  - (b) (5%) Is  $f$  continuous at 0? Explain your answer as well.
  - (c) (8%) Is  $f$  differentiable at 0? Explain your answer as well.
  
2. Let  $f(x) = \ln x, x > 0$ .
  - (a) (7%) Find the Taylor Series for  $f(x)$  centered at 1.
  - (b) (8%) Find the interval of convergence of the power series in (a).
  - (c) (5%) Find the value of the infinite series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ .
  - (d) (5%) Approximate  $\ln(1.1)$  by the 4<sup>th</sup> Taylor Polynomial for  $f$  centered at 1.
  
3. Evaluate the following integrals.
  - (a) (6%)  $\int \frac{1}{x + \sqrt{x}} dx$
  - (b) (6%)  $\int_0^{\infty} x^3 e^{-x^2} dx$
  - (c) (8%)  $\int_0^1 \frac{x}{(x+1)^2(x^2+1)} dx$
  
4. Let  $f(x, y) = \frac{8y}{1+x^2+y^2}$  and  $C$  be the level curve of  $f(x, y)$  at the level  $f(x, y) = 2$ .
  - (a) (5%) Find the equation of  $C$ .
  - (b) (7%) Find the equation the tangent line at the point  $(\sqrt{3}, 2)$  on level curve  $C$ .
  - (c) (8%) At the point  $(\sqrt{3}, 2)$  on the level  $C$  what is the direction of the greatest rate of increasing of  $f(x, y)$ ?
  
5. (15%) Evaluate  $\iint_R e^{-xy} dA$ , where  $R$  is the region bounded by  $x=1, x=3, xy=1$ , and  $xy=4$ .